# Innovative Techniques for Quantifying Bubble-Particle Attachment in Flotation: Insights from Experimental, Modelling, and Numerical Investigations

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**EXTENDED ABSTRACT**

**Background and Challenges**

Bubble–particle attachment is a fundamental subprocess in flotation, governed by intricate physicochemical interactions (Nguyen and Schulze, 2004; Zheng et al., 2025a). To explore the attachment process more deeply, researchers have employed a variety of specialised techniques, such as induction timer (Albijanic et al., 2010), bubble–particle attachment angle test (Zheng et al., 2024), Miller timer (Verrelli et al., 2014), and atomic force microscopy (Nguyen et al., 2003). These methods enable detailed probing of induction time, liquid film thinning and rupture, attachment kinetics, and interfacial forces, among other aspects, in the attachment mechanism. However, they are typically conducted in systems isolated from flotation itself, which limits their capacity to relate microscopic interactions to macroscopic flotation performance directly. In this context, the single-bubble flotation apparatus (see **Figure 1**a) offers a rare advantage: it serves as an idealised platform that can simultaneously simulate simplified flotation and correlate flotation rate constants with micro-interfacial parameters such as collision efficiency, attachment efficiency and induction time. Nevertheless, focusing exclusively on attachment using the classical single-bubble flotation approach is challenging, as the influences of collision and detachment cannot be entirely neglected during experiments. In classical single-bubble experiments, detachment is often assumed negligible for fine, hydrophobic particles under quiescent flow, yet collision remains an unavoidable prerequisite for attachment. Since collision efficiency is inherently low, particularly under turbulent or high pulp density conditions, the interference of poor collision can lead to significant underestimation of intrinsic attachment efficiency. This is especially problematic in the flotation of ultrafine particles, where weak hydrodynamic interactions make effective collision challenging, masking the actual behaviour of the attachment step. Therefore, the interference from collision efficiency is likely to cause its effect to be severely underestimated when studying the attachment process. In other words, how can one design a flotation experiment where bubble–particle attachment is the dominant subprocess, while the effects of collision and detachment can be safely neglected?

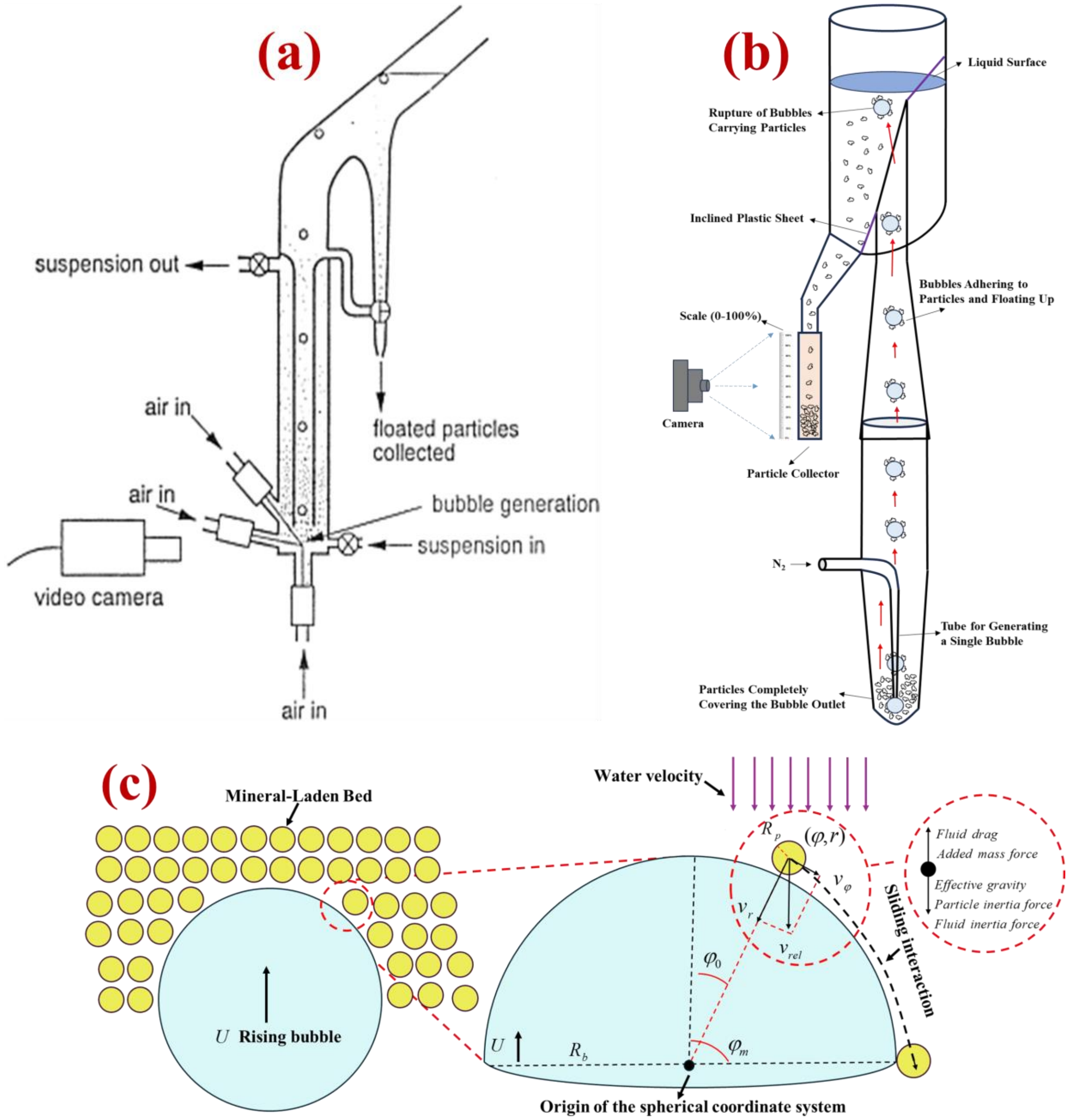


**Figure 1.** (a) Classic single-bubble flotation test device (Hewitt et al., 1995; Nguyen et al., 1998). (b) An innovative experimental technique is proposed in this study. (c) The basic physics mechanism of particle motion on the bubble surface in our novel technique.

### The Innovative Technique Proposed in Our Research

To overcome the limitations associated with collision interference in conventional single-bubble flotation tests, we propose a novel technique that employs rising bubbles interacting with a stationary particle-laden bed (see **Figure 1b** and **1c**) . This configuration enables the systematic evaluation of bubble–particle attachment processes, assuming near-unity collision efficiency. By constraining the spatial distribution of particles within a shallow, well-characterised bed, and allowing individual bubbles to interact with multiple particles in a controlled trajectory, the influence of stochastic collision probability is effectively eliminated. Furthermore, a mathematical framework and models were developed to interpret the experimental results, quantify the sliding attachment kinetics, and back-calculate the induction time and attachment efficiency.

Compared to traditional single-bubble flotation or attachment measurement techniques, this new configuration offers several distinct advantages. First, by minimising uncertainty in the collision process, the method enables more accurate quantification of attachment efficiency and induction time. Second, the setup allows for systematic variation of experimental parameters—such as bubble size, velocity, particle properties, or electrolyte composition—facilitating detailed studies on the interfacial mechanisms governing bubble–particle attachment.

**Experimental Investigations**

**Aims and targets:** Particle shape has played an essential role in flotation (Nguyen and Schulze, 2004; Ulusoy, 2023; Xia, 2017; Zheng et al., 2025d). The impact of mineral particle shape on flotation efficiency has been extensively studied for various minerals (Guven et al., 2015; Rahimi et al., 2012; Ulusoy et al., 2004; Vizcarra et al., 2011; Zheng et al., 2024), among others. Additionally, previous experimental studies have indicated that particle shape can influence bubble–particle attachment efficiency and induction time (Hassas et al., 2016; Ma et al., 2023; Verrelli et al., 2014) . However, two critical limitations remain unresolved. First, most of these findings are based on statistical analyses of attachment-related parameters and lack direct correlation with flotation rate or recovery, making it difficult to establish a mechanistic link between attachment kinetics and overall flotation performance. Second, the underlying colloid-chemical mechanisms through which particle shape affects attachment remain poorly understood. Addressing these gaps, our experimental study aims to investigate the attachment process using the newly developed particle-laden bed technique, which enables more direct quantification of attachment dynamics. This experimental investigation also serves to validate the theoretical and experimental applicability of the method and provide an example of how to conduct a topic researcher on attachment using this technique.

**Experimental steps:** We employed our proposed technique, together with an associated theoretical model, to systematically study the effect of particle shape on the bubble–particle adhesion process. Two types of test particles were selected: spherical glass beads and irregularly fractured beads, both derived from the same base material and sharing a narrow size range (+150–212 µm). Surface preparation involved cleaning, drying, and esterification using 1-octanol to render the particles hydrophobic. The degree of hydrophobicity was quantified using the Washburn capillary rise method to determine contact angles. Particle shape and angularity were characterised through image analysis, while surface charge (zeta potential) was measured across varying salt concentrations using microelectrophoresis. Micro-flotation tests were carried out in both deionised water and NaCl solutions of varying concentrations using our particle-laden bed apparatus. This allowed for direct comparison of flotation recoveries and attachment characteristics between the two particle shapes.

**Results and findings:** To isolate the shape effect, both particle types were rendered hydrophobic via identical esterification procedures. Contact angle measurements using the Washburn method verified that surface hydrophobicity was comparable, ensuring that any differences in flotation behaviour were attributable to shape alone. Flotation tests using the particle-laden single-bubble apparatus demonstrated that irregular particles consistently exhibited higher flotation recoveries than spherical ones. The extended theoretical analysis revealed that these differences were driven by higher attachment efficiency and shorter induction times for the broken particles, indicating their geometry-enhanced attachment behaviour (see **Figure 2**). Further investigations explored the influence of NaCl concentration. The flotation recovery of spherical beads increased significantly with salt concentration, while the broken particles showed less sensitivity. Zeta potential measurements and corresponding calculations of electric double layer (EDL) disjoining pressure indicated that increasing ionic strength reduces electrostatic repulsion, especially at 0.1 M NaCl, aligning well with the observed trends in flotation recovery, rates, induction time and attachment efficiency.

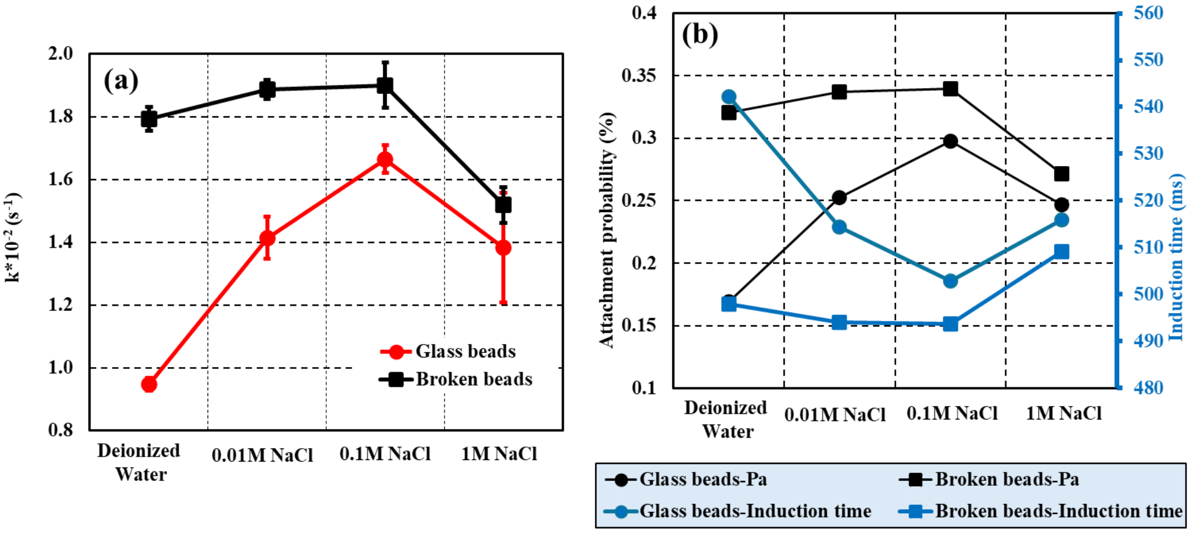

**Figure 2**. Comparison of the key parameters (a: flotation rates; b: attachment probability and induction time) for the flotation of glass beads and broken beads versus salt concentration (Zheng et al., 2025c).

**Mechanism analysis:** The variations in induction time, attachment efficiency, and flotation rate of the two particle shapes with increasing salt concentration were comprehensively analysed in conjunction with the trends in repulsive EDL disjoining pressure. Using a hypothetico-deductive approach, we assumed that if particle shape had no influence on colloidal interactions during attachment, particularly the repulsive disjoining pressure, then the differences in flotation rate and attachment parameters between the two shapes under different salt concentrations should remain the same as those observed in deionised water. However, this assumption was clearly inconsistent with the experimental results. This discrepancy suggests that in deionised water, irregular particles interact with bubbles in a way that leads to more pronounced compression of the electric double layer compared to spherical particles. As a result, the two shapes respond differently to increasing salt concentration. This confirms that the influence of particle shape on attachment is related to the EDL disjoining pressure. It is worth noting that since EDL forces are primarily determined by the interfacial charge between bubble and particle and are largely independent of the intrinsic properties of the particle itself, we approximate the EDL interaction strength to be similar for both shapes. Therefore, the difference in disjoining pressure is mainly attributed to differences in effective contact area (see Eq. (1)). At present, we are unable to determine whether van der Waals and hydrophobic forces vary with particle shape, and thus, these forces are not discussed in this work.

Consequently, these results reveal that irregular particles, with sharp edges and elongated features, achieve more effective contact with bubbles (larger contact area with liquid film), reduced EDL repulsive disjoining pressure (see Eq. (1)), resulting in faster film thinning (see Eq. (2)), and thus decreasing the induction time (see Eq. (2)), increasing attachment efficiency (see Eq. (3)), finally enhanced flotation performance (see **Figure 3**). This confirms that particle morphology has a direct impact on interfacial interaction dynamics based on repulsive EDL disjoining pressure.

$$\frac{Force}{Unit\ area} = Pressure \tag{1}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t_{thin}} = -\frac{2h^3}{3\mu {R_f}^2}\left(P_\sigma - \Pi\right) \tag{2}$$

In this context, $h$ represents the thickness of the liquid film, $t_{thin}$ is the time for liquid film thinning, $\mu$ denotes the viscosity of the liquid, $R_f$ is the radius of the film, $P_\sigma$ stands for the capillary pressure, $\Pi$ is disjoining pressure.

$$P_{at} = P_f \cdot P_r \cdot P_{tpc} \tag{3}$$

where the first step is the thinning of the liquid film to a critical thickness, with a probability of $P_f$. The second step is the rupture of the liquid film upon reaching the critical thickness, forming a three-phase contact (TPC) line (TPCL), with efficiency of $P_r$. The third step is the expansion of the TPCL from the critical radius to form a stable wetting perimeter, with a efficiency of $P_{tpc}$. These three processes determine the attachment efficiency $P_{at}$, as shown in Eq.(3) (Nguyen et al., 1998).

In summary, these experimental investigations highlight three aspects of significance: (1) **Key mechanism identified:** Particle shape influences induction time and attachment efficiency primarily through variations in the electric double layer (EDL) disjoining pressure and the particle–bubble contact area. (2) **Technical highlights:** The interference from other subprocesses (e.g., collision and detachment) was effectively minimised using the particle-laden bed configuration. The system enables reproducible, shape-dependent analysis of bubble–particle attachment under controlled conditions. (3) **Practical significance:** This innovative developed technique demonstrates strong applicability for investigating fundamental attachment mechanisms in flotation, as observed in experimental investigations. Furthermore, the research framework established in this study—namely, using the newly developed technique to determine flotation rate, followed by model-based back-calculation of attachment efficiency and induction time to quantify attachment kinetics. With the Stefan–Reynolds theory serving as a theoretical bridge—offers a generalizable approach, this methodology enables the construction of a comprehensive relationship linking flotation kinetics, attachment parameters (efficiency and induction time), and underlying colloidal interaction mechanisms by incorporating the effects of salt concentration on electric double layer (EDL) interactions and disjoining pressure. This methodology is applicable to other flotation systems and interface-dominated processes.

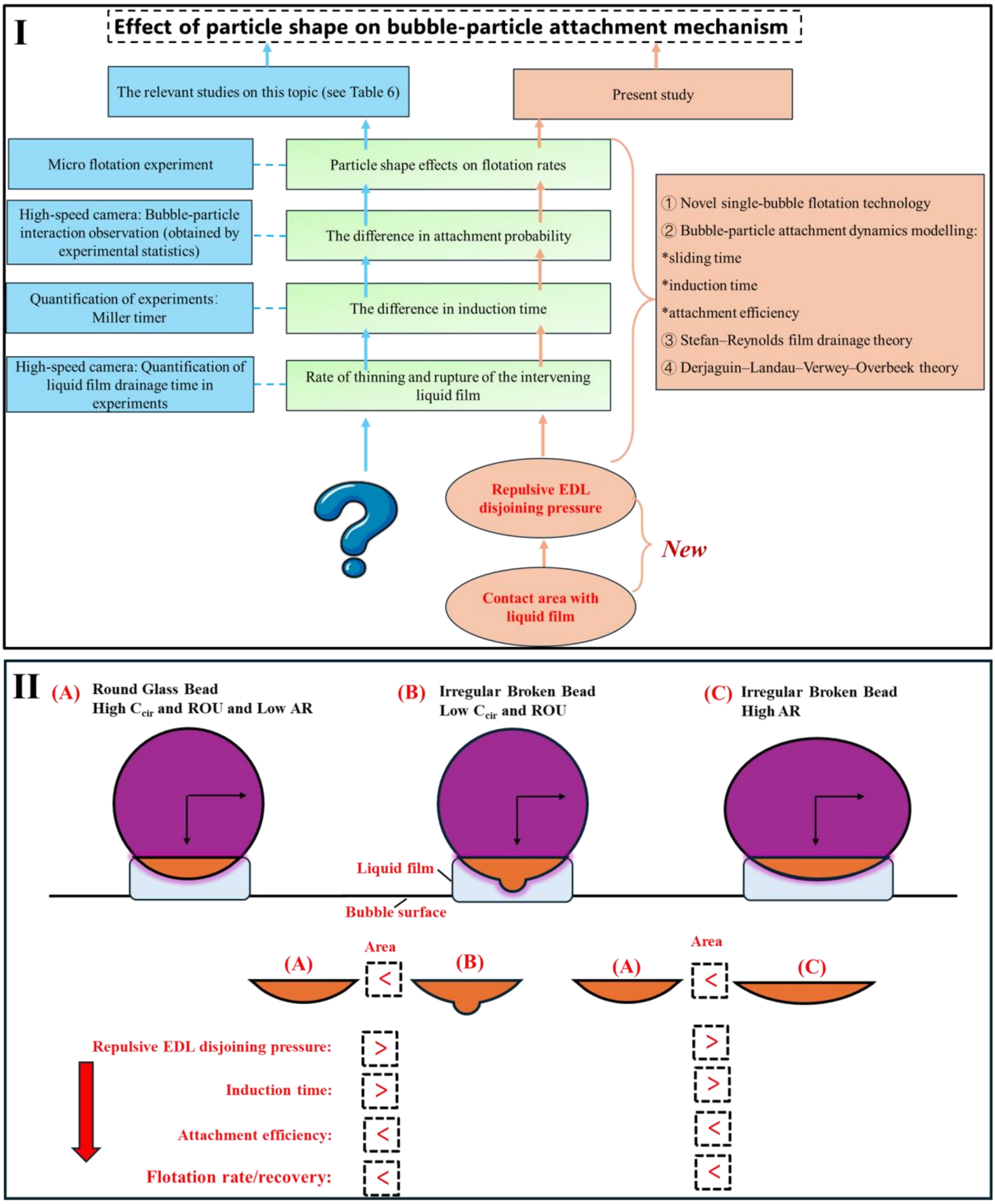


**Figure 3.** (I) Comparison of the mechanism analysis in the present study with previous studies on bubble-particle attachment for shape effects; (II) Mechanism diagram of how particle shape affects particle-bubble attachment: comparison of contact area differences between round glass bead (A) and irregular broken bead (B and C) with the intermediate liquid film (Zheng et al., 2025c).

## Modelling Investigations

Our primary objective is to validate the effectiveness and applicability of the proposed apparatus, while establishing a theoretical and modelling framework that can serve as a practical tool for researchers investigating bubble–particle attachment phenomena. In this system, a single bubble rises through a particle-laden bed, and the sliding time of individual particles along the bubble interface—measured from the point of initial contact to detachment or attachment—is quantitatively evaluated using both analytical and numerical models. The analytical (non-inertial) model refines the foundational formulation proposed by Nguyen (1993), accounting for Stokes flow conditions and integrating a shape-dependent drag coefficient. In parallel, an inertial model is constructed based on an extended form of the Basset–Boussinesq–Oseen (BBO) equation, which incorporates unsteady flow and history effects. This model is solved numerically using a fourth-order Runge–Kutta scheme. To account for particle shape and Reynolds number effects, both models implement drag coefficient approximations representing Stokesian and non-Stokesian flow regimes. The key step of modelling is shown in **Figure 4**.

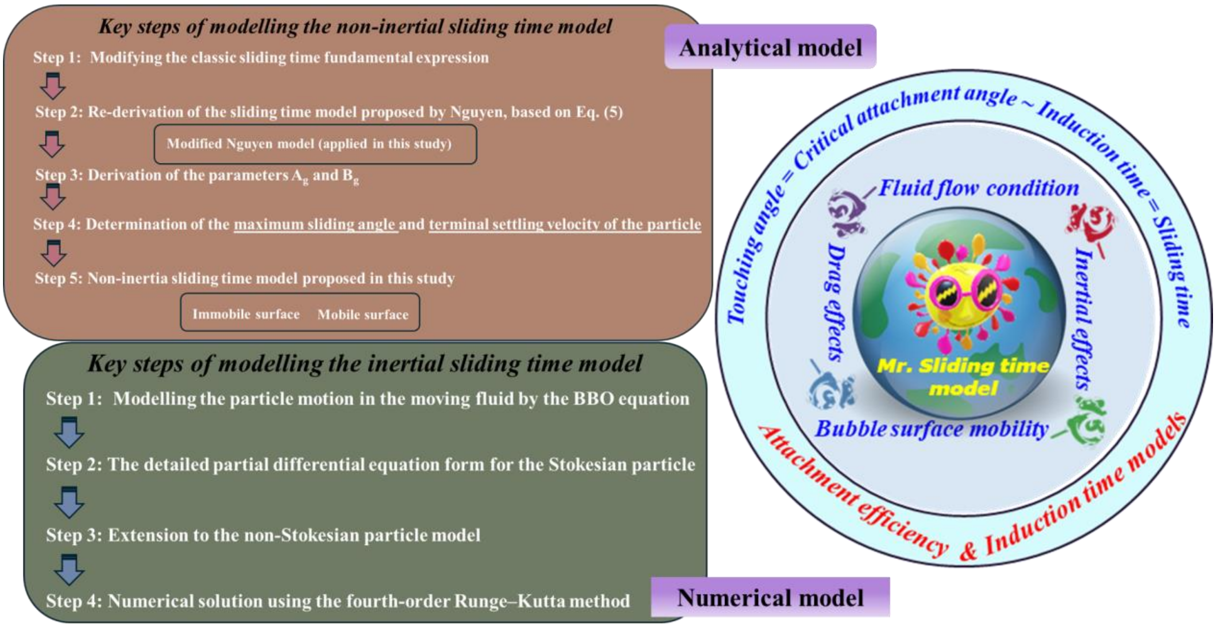


**Figure 4.** The key steps of modelling of particle sliding time during the bubble's passage through the particle-laden bed, based on both non-inertial analytical and inertial numerical approaches. The figure also shows how the analysis links sliding time to attachment efficiency and induction time, highlighting the key scientific questions involved in the bubble–particle attachment process.

The resulting sliding time serves as a mechanistic bridge between microscale interfacial processes and macroscale flotation behaviour. Specifically, it enables the back-calculation of induction time and attachment efficiency, offering a physically grounded framework for correlating flotation kinetics with underlying particle–bubble interaction dynamics. **Figure 5** illustrates the structure and capabilities of the sliding time model, highlighting its role in connecting micro- and macro-scale perspectives in flotation. The proposed framework allows for reversible interpretation—using flotation data to infer attachment kinetics, and vice versa—thus advancing both predictive modelling and mechanistic understanding.

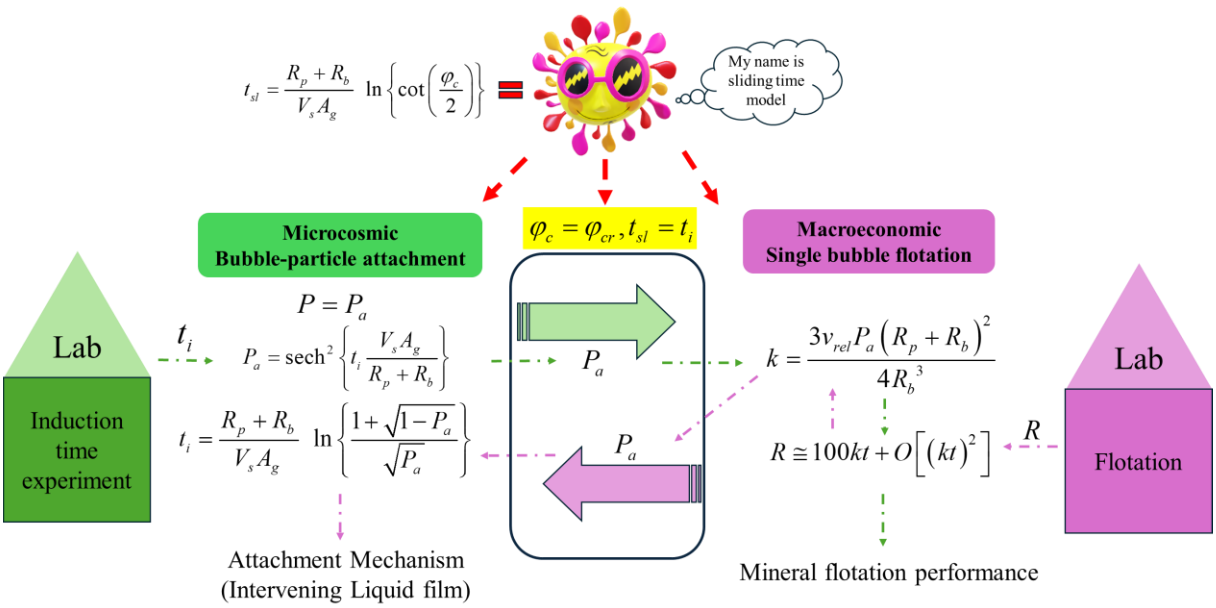


**Figure 5.** The linkage between macroscopic flotation kinetics and the microscopic bubble-particle attachment process was constructed using the non-inertial sliding time model. Flotation rate constants derived from the flotation process can be used to predict induction time via attachment efficiency as a bridge, enabling analysis of the attachment mechanism, particularly the dynamics of the intervening liquid film. Conversely, induction time measured from experiments can be used to predict flotation rate constants, allowing an evaluation of mineral flotation performance. These analyses are based on our novel single-bubble flotation apparatus, which directly correlates flotation outcomes with attachment efficiency, minimising interference from other subprocesses (Zheng et al., 2025b).

**Numerical Investigations**

**Some key points of bubbles through the particle-laden bed.** Our combined numerical and analytical investigations confirmed that the particle sliding time on the bubble surface is significantly influenced by bubble size, while the dependence on particle size varies across the five approximate models evaluated (see **Figure 6**). Among the key findings, fluid drag was identified as the dominant factor affecting sliding time within the range of model validity, whereas the contribution of particle inertia was found to be relatively minor under typical flotation conditions. In addition to predicting sliding duration, the models effectively describe the detailed kinematic behaviour of particles along the bubble interface. These include the time-resolved profiles of tangential and radial velocity, as well as the horizontal and vertical displacements of particles during the interaction (see **Figure 7**). Notably, these parameters are strongly dependent on the particle–bubble contact angle at the moment of collision, highlighting the critical role of initial collision orientation in determining attachment dynamics. The ability to resolve such detailed motion trajectories provides a powerful framework for linking geometric and hydrodynamic conditions to attachment outcomes. This reinforces the utility of the proposed sliding time model in interpreting experimental observations and guiding the design of flotation strategies and devices.

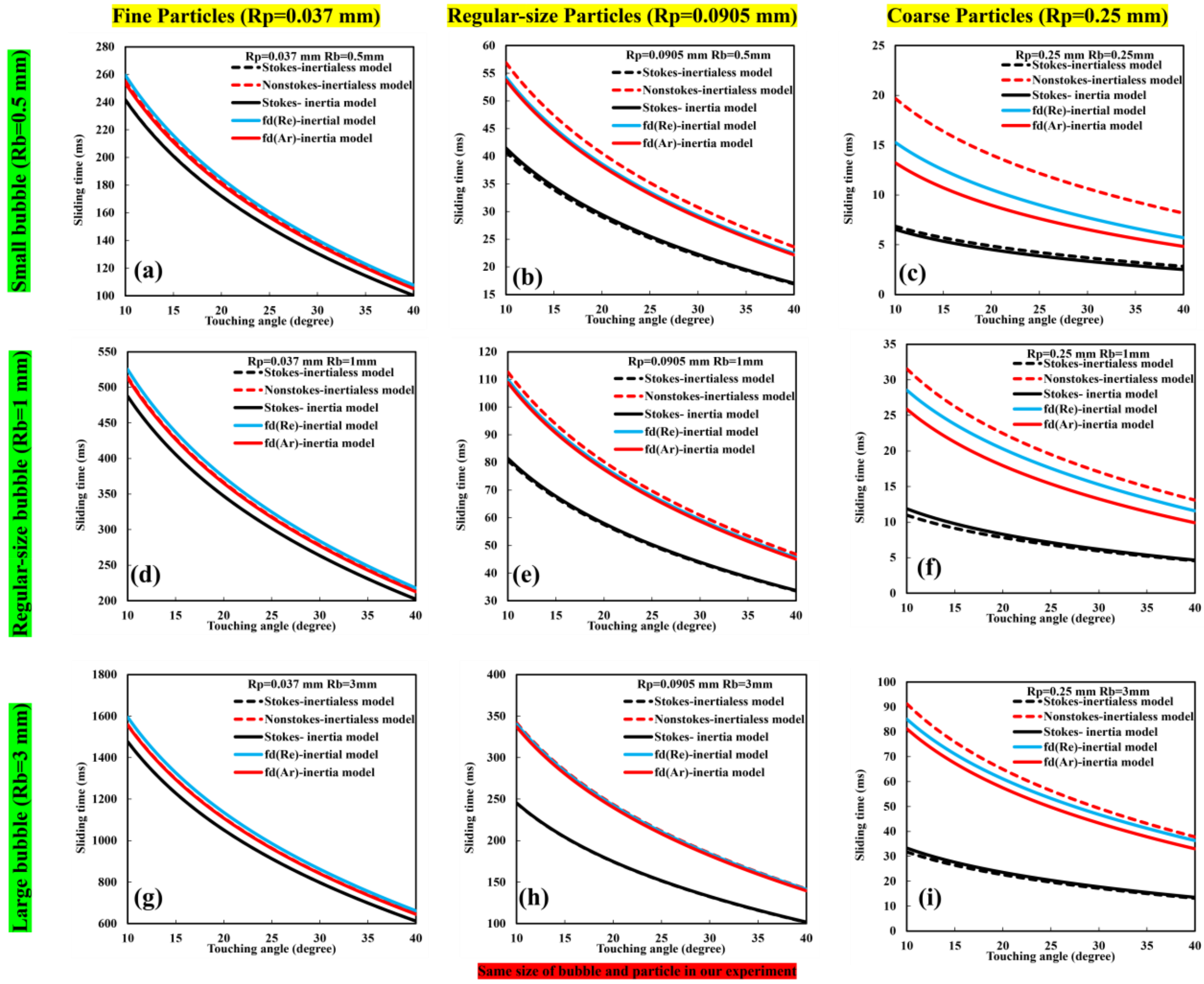


**Figure 6**. Variation of sliding time with touching angle using five approximate models (for different particle and bubble sizes). The 3×3 matrix of plots illustrates the analysis results for various bubble and particle sizes. The columns represent fine, medium-sized, and coarse particles under the same bubble size from left to right. From top to bottom, the rows represent

small bubbles, medium-sized bubbles, and large bubbles, respectively, for the same particle size (Zheng et al., 2025b).

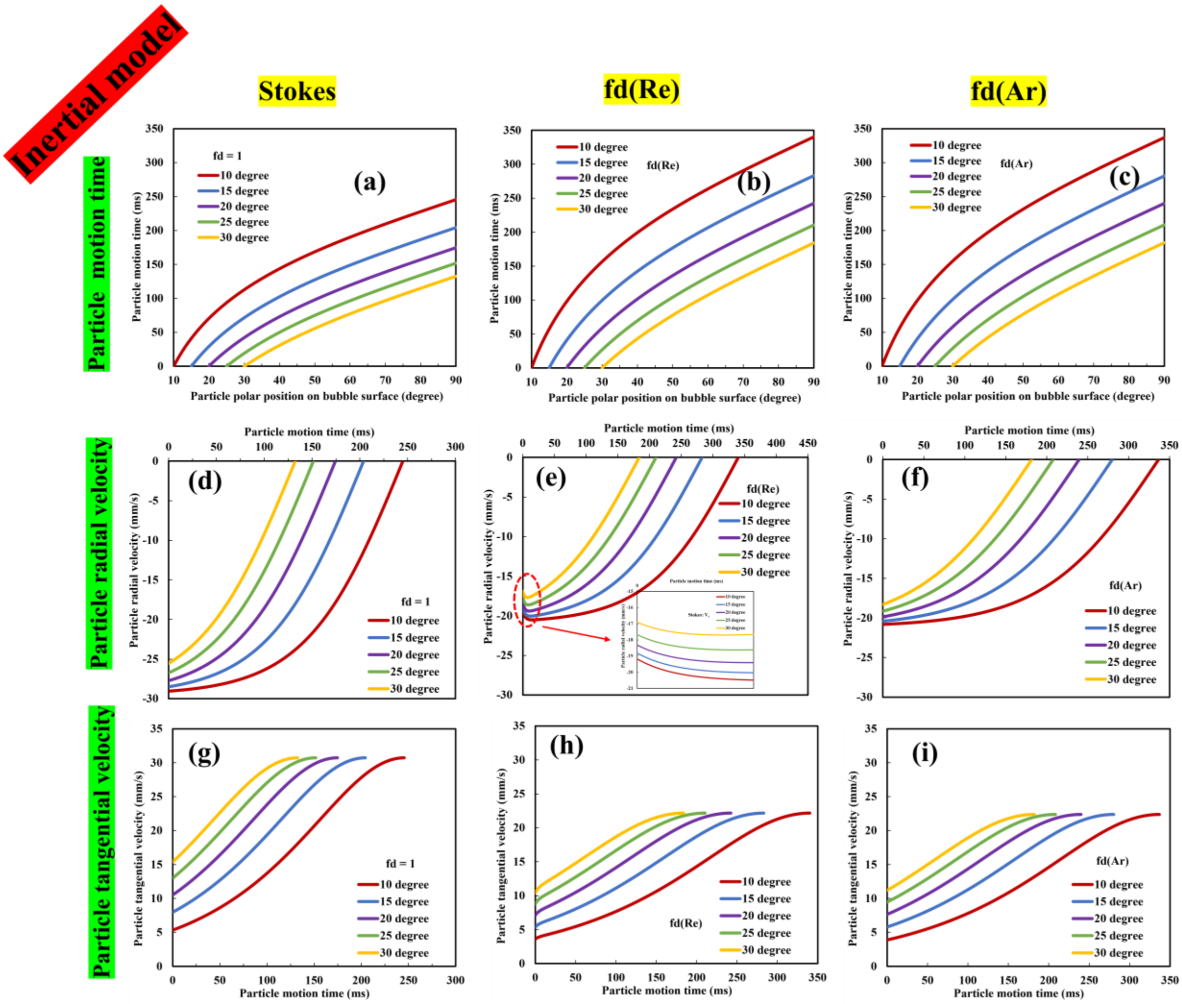


**Figure 7**. The kinematic parameters of particle circular motion from the touching angle position to the 90° position, including motion time, radial velocity, and tangential velocity, are numerically calculated based on experimental conditions: particle radius = 0.0905 mm and bubble radius = 3 mm. The results are presented in a 3×3 grid of plots, where the horizontal axis (left to right) represents the time of three different approximation schemes: Stokes, $f_d\left(\mathrm{Re}\right)$, and $f_d\left(Ar\right)$ for the inertial numerical simulations, while the vertical axis (top to bottom) corresponds to various kinematic parameters (Zheng et al., 2025b).

**Particle shape effects on sliding kinetics based on shape factor.** More importantly, numerical simulations incorporating the nonspherical settling velocity and nonspherical drag force model demonstrated that particle shape (aspect ratio and sphericity) strongly influences sliding parameters, attachment efficiency, settling velocity, and drag coefficient. These findings provide a physical and fluid-dynamic explanation of how particle shape influences flotation, particularly through the sliding mechanism involved in the attachment process (see **Figure 8**).

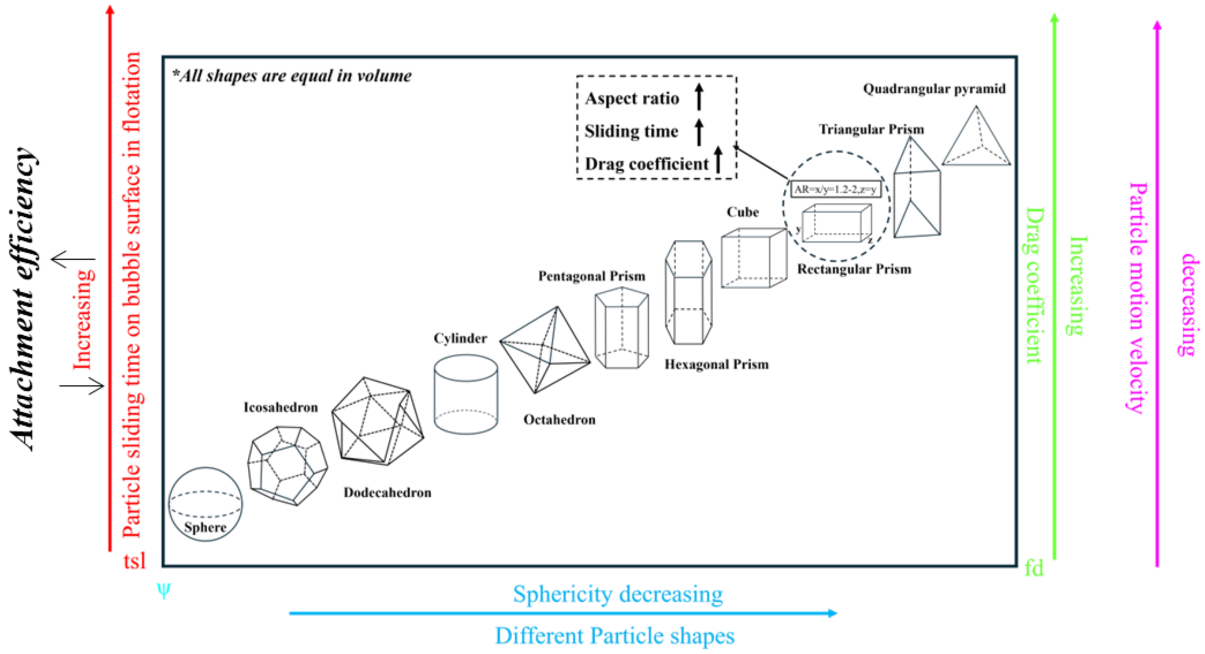


**Figure 8.** The relationships among sliding time, adhesion efficiency, and drag coefficient under different sphericities and aspect ratios (AR). It should be noted that all particle shapes have the same equivalent volume diameter. The numerical analysis is based on the model of the present system, with the influence of non-spherical particles on sliding parameters evaluated by incorporating a drag model that accounts for sphericity. This study does not consider the actual geometric features of particle shapes or the contact orientation between the particle and the bubble, which may affect the tangential and radial components of the sliding process. These aspects involve more complex three-dimensional modelling and are beyond the scope of the present study, which focuses solely on the effects of shape factor differences.

**Limitations and further work**

The novel technique and theoretical framework developed in this study provide a valuable platform for investigating bubble–particle attachment and understanding the role of individual variables during this flotation subprocess. However, the current work primarily emphasises conceptual device design and theoretical development under controlled, idealised conditions. As such, several limitations must be acknowledged to ensure scientifically rigorous interpretation of the findings and cautious extrapolation to real flotation systems.

**Limitation in extrapolating to real flotation environments:** The experimental design in this study assumes a bubble–particle collision efficiency of 100%. However, in practical flotation systems—such as mechanical cells or flotation columns—overall performance is determined by the combined effects of both collision and attachment efficiencies. Therefore, a variable that improves attachment efficiency in isolation does not necessarily lead to enhanced flotation performance under real operating conditions. Caution is thus required when interpreting the influence of such variables in practical applications.

**Model limitations and targets for improvement:** The present study introduces a novel technique and theoretical framework under controlled micro-scale conditions (single-bubble micro-flotation). However, several key limitations, such as the potential effects of flow regime, turbulence, bubble swarms, surfactants, particle morphology, microhydrodynamics, and colloidal forces, must be acknowledged and considered when extending our approach to industrial flotation environments or exploring novel flotation equipment.

Future development of models and scale-up flotation systems based on the proposed framework should systematically incorporate the factors mentioned earlier. This will enhance model reliability and support the design of high-efficiency flotation equipment aimed at improving bubble–particle collection efficiency.

## ACKNOWLEDGMENTS

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